

Course Type	Course Code	Name of Course	L	T	P	Credit
DE	NMED528	Navier-Stokes solvers	3	0	0	3

Course Objectives

1. To familiarize the students with the classical approaches of numerically solving the Navier- Stokes (or N-S) equations of motion and aid them develop a solid foundation on the solvers. The students mostly with prior background of dealing with a single ODE/PDE will have detailed flavour of dealing with systems of coupled non-linear PDEs.
2. Step-by-step illustration of the basic algorithms and computer codes for two-dimensional Navier-Stokes solvers using finite volume (for sequential SIMPLE solver) and finite element (for coupled solver) methods.

Learning Outcomes

1. The students will be confident enough to write Navier-Stokes codes on structured mesh in two-dimensions.
2. The students will appreciate the differences between FVM and FEM approaches with respect to a given flow problem in two-dimensions.
3. The students will be enlightened on different advanced linear equation solvers (eg., non- stationary iterative solvers) that are extensively used for solving the asymmetric flow matrix obtained using FEM.

Unit No.	Topic to be Covered	Lecture Hours	Learning outcome
1	MODULE I The Navier-Stokes equations of motion in primitive variables – various couplings, mathematical behaviour and properties, applicability, derivation of equations, Stokes equations, Streamfunction-vorticity equations, velocity-vorticity equations, pressure Poisson equation	6	The students will learn the method of derivation of differential form of the N-S equations starting from an integral form. This introductory module will also familiarize them with different variants of the N-S equations
2	MODULE II Need for methods for approximate solutions of Navier-Stokes equations, strong and weak forms of a differential equation	1	Realization of the importance of numerical methods in solving the N-S equations
3	MODULE III The classification of primitive variable based Navier-Stokes solvers – the sequential and coupled solvers, collocated and staggered arrangement of primitive variables, the first sequential MAC solver of Harlow and Welch using FDM	6	Understanding of the difficulties encountered while dealing with the N-S equations numerically. Introduction to the first primitive variable based N-S solver

4	MODULE IV Discretization of the convection term via central difference scheme, power-law scheme, hybrid scheme, first order upwinding scheme, second order upwinding scheme, QUICK scheme, discretization of the diffusion term, the need for upwinding, the odd-even decoupling of pressure	6	Assessment of various competing discretization schemes for approximating the convection term
5	MODULE V The SIMPLE family of sequential Navier- Stokes solvers on staggered structured mesh: complete demonstration of the algorithm using FVM for two-dimensional lid-driven cavity problem on u -mesh, v -mesh and p - mesh	8	Complete understanding of the classical SIMPLE solver on FVM staggered mesh
6	MODULE VI LBB compatibility criterion, the coupled Navier-Stokes solver using Galerkin FEM or GFEM: derivation of element level matrices, vectors, implementation of boundary conditions, assembly of element level matrices and vectors using Q1Q0 elements	8	Generation of the linearized asymmetric global matrix equations for flow problems under the framework of GFEM using Q1Q0 elements
7	MODULE VII The limitations of GFEM and need for Petrov-Galerkin (PGFEM) formulation, PGFEM on collocated mesh using Q1Q1 elements, SUPG and PSPG stabilizations, derivation of element level matrices, vectors, implementation of boundary conditions, assembly of element level matrices and vectors using Q1Q1 elements	7	Need for stabilized FEM formulation for Q1Q1 elements, differences between GFEM and PGFEM

Total 42 hours

Text Books:

1. H.K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics*, Pearson Education Limited, 2nd Edition, 2007.
2. J. Donea and A. Huerta, *Finite Element Method for Flow Problems*, John Wiley & Sons, Ltd., 2003.

References:

1. S.V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, 1st Edition, 1980.
2. Richard H. Pletcher, John C. Tannehill and Dale A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, 3rd Edition, 2012.
3. Joel H. Ferziger and M. Peric, *Computational Methods for Fluid Dynamics*, Springer, 3rd Edition, 2002.